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THERMAL CONDUCTIVITY OF A THREE-DIMENSIONAL BODY WITH A ROUGH INCLUSION

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Thermal conductivity of a three-dimensional body with a rough inclusion is studied. The problem is reduced to the problem of conjugation of harmonic functions and its exact solution is constructed. Distributions of temperature and heat-flux density near distinctions of the interface between media are found. Results are illustrated by a numerical example.

We study the thermal conductivity of an infinite body with a heat-conducting inclusion which occupies a finite simply connected region V_1 and is joint to a matrix by the conditions of an ideal thermal contact. There are a finite number of roughnesses of the type of nonintersecting closed smooth singular lines (sets of angular points) and conical points on the boundary surface of the inclusion. The temperature field of this composition is determined under outer thermal effects (heat flux at infinity, concentrated heat sources, uniform heating, etc.).

The temperature fields of the matrix (body) and inclusion T_j ($j = \overline{0, 1}$) are harmonic functions [1]. Thus, we must solve the problem of conjugation of harmonic functions by the conditions of an ideal thermal contact [1] on a rough surface. A value of the subscript $j = 0$ corresponds to values of the matrix (body), which occupies the region $V_0 = R^3 \setminus V_1$ (R^3 is a three-dimensional space), and $j = 1$ corresponds to values of the inclusion V_1. This problem of conjugation of harmonic functions in the regions with piecewise-smooth boundaries is solved in [2] and, based on this work, a method for solving the problem of thermal conductivity of a body with a rough inclusion is suggested. As a result, the problem is reduced to the integral equation with a polar kernel. In [2] it is shown that this equation is usually solvable, and its exact solution is obtained by the method of successive approximations.

It is found on the basis of the approach suggested that distributions of the temperature field and components of the vector of heat-flux density near the singular line of the interface between media in the matrix have the form

$$T_0 = \rho^m (k_1 \cos m\theta + k_2 \sin m\theta)/(\gamma - 1) + O(\rho^{m+1}),$$

$$q_{0\rho} = \lambda_0 m \rho^{m-1} (k_1 \cos m\theta - k_2 \sin m\theta)/(\gamma - 1) + O(1),$$

$$q_{0\theta} = \lambda_0 m \rho^{m-1} (k_1 \sin m\theta + k_2 \cos m\theta)/(\gamma - 1) + O(1),$$

$$q_{0s} = O(1),$$

where λ_j are the coefficients of thermal conductivity of the matrix ($j = 0$) and the inclusion ($j = 1$); $\gamma = \lambda_0/\lambda_1$; $m = \pi/(2\pi - \omega)$; ω is the value of the expansion angle of the interface at a point of the singular line; k_1 and k_2 are the heat-flux intensity factors; ρ, θ, and s are the coordinates of the local region of the singular line [2].

We present distributions of the temperature field and heat flux, for example, in the matrix near the apex of the conical interface with a circular directrix in the form

$$T_0 = \rho_1^{m_k} k_0 P_{m_k} (\cos \theta_4) + O(\rho^{m_k+1}), \quad q_{0\rho_1} = -\lambda_0 m_k \rho_1^{m_k-1} k_0 P_{m_k} (\cos \theta_4) + O(1),$$

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$$q_{0\theta_1} = -\lambda_0 \rho_1^{m_k - 1} k_0 dP_{m_k}(\cos\theta_4)/d\theta_4 + O(1), \quad q_{0s_1} = O(1),$$

where $m_k \in (0, 1)$ is the root of the characteristic equation $P_{m_k}(\cos\theta_4) = 0$ at a value of $\theta_1 = \pi/2$ which determines a circular conical surface; $P_{m_k}(\cos\theta_4)$ is the Legendre function; $\theta_4 = \theta_1 + \beta$, $\tan\beta = d$; the quantity d is determined by the geometry of the conical surface; ρ_1, θ_1, and s_1 are the coordinates of the local region of the conical point [2].

The distributions in the inclusion as well as in the case of another shape of the directrix of the conical surface have a similar form. Letting the quantity γ tend to zero (infinity) in such distributions, we obtain distributions for the case of an isothermal (thermally insulated) inclusion.

Based on the method suggested, we studied the thermal conductivity of an infinite body with a circular conical inclusion bounded by a surface S:

$$x^2/a^2 + y^2/a^2 - z^2/c^2 = 0, z = c.$$

This interface between media has a singular line L formed by the base of the cone $z - c = 0$ and its side surface $x^2/a^2 + y^2/a^2 - z^2/c^2 = 0$ and the conical point $O_1(0, 0, 0)$. It is assumed that the composite is subject to uniform heating by a value of ΔT. To solve the problem completely, we added the conditions of thermal balance of the regions involving distinctions of the interface between media.

It was assumed in the numerical analysis that the thermal characteristics correspond to a composite of the steel–brass type. As a result, we found the dependence of relative values of the heat-flux intensity factors $k_0/k_p^{(0)}$ ($p = \overline{1, 2}$) near the singular line on the value of the angle ω of generatrix inclination with respect to the base. Here $k_1^{(0)}$ and $k_2^{(0)}$ are equal to the values of the intensity factors at a zero angle of inclination of the cone generatrix to the base. With an increase of the angle of inclination ω the quantity $k_1/k_1^{(0)}$ increases, whereas $k_2/k_2^{(0)}$ decreases. The singularity ρ^{m-1} of the heat-flux density near the singular line becomes weaker as the angle ω of generatrix inclination with respect to the base increases from 0 to $\pi/2$.

In the example presented, for the same singularity the intensity factors remain constant in the circumferential direction of the singular line (circumference); such a situation is also observed for a local flow near the conical point. This is caused by both the geometric symmetry (circular cone) and the symmetry of the thermal load (uniform heating).

Thus, it is found that the presence of singular lines (sets of angular points) and conical points on the interface between a three-dimensional body and a foreign inclusion leads to disturbance of the temperature field. The components of the vector of heat-flux density acquire exponential singularities. Thin heat-conducting foreign interlayers (thin curvilinear inclusions) are responsible for singularities of the order of 0.5 of the disturbed local heat flux.

The intensity of the heat-flux density near distinctions of the interfaces between media is characterized by both the order of the singularity and the heat-flux intensity factors. A determining role is played by the order of the singularity, and the intensity factors are an important characteristic for the same order of singularity.

NOTATION

T_0, temperature in the matrix near distinctions of the interface; $q_{0\rho}$, $q_{0\theta}$, and q_{0s}, components of the vector of heat-flux density in the matrix in the local coordinates ρ, θ, and s near the singular line; $q_{0\rho_1}$, $q_{0\theta_1}$, and q_{0s_1}, components of the vector of heat-flux density in the matrix in the local coordinates ρ_1, θ_1, and s_1 near the conical point.

REFERENCES

1. A. V. Luikov, *Heat and Mass Transfer* [in Russian], Moscow (1978).
2. I. T. Denisyuk, *Izv. Vyssh. Uchebn. Zaved., Matematika*, No. 4, 29–35 (2002).